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Form Factors of K_{13} Decay and Kaon Production by High-Energy Neutrinos.

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Summary. — The cross-section of the kaon-production reaction by high energy neutrinos, $\nu+p\rightarrow p+l^-+K^+$, is computed in the peripheral approximation. Numerical estimates are made with specific form factors derived from the dispersion theory. The cross-section depends on such a dynamical model, hence it can serve as a criterion of the model.

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1. – The form factors of K_{13}^+-decay have been studied as a function of s, the square of the four-momentum transfer, by several authors in dispersion-theoretical approach ([1-3]). Since the range of s is rather narrowly limited in the decay reaction ([4]), the behaviour of the form factors is not fully revealed in the decay process. It is hoped that future experiments with high-energy neutrinos will provide a means to study the form factors in a wider range of s. We point out here that the reactions of charged-kaon production by neutrinos,

$$\nu+p\rightarrow p+l^-+K^+\,, \tag{1}$$

$$\nu+n\rightarrow n+l^-+K^+\,, \tag{2}$$

are particularly fit for such a purpose.

([1]) S. W. MAC DOWELL: *Phys. Rev.*, **116**, 1047 (1959).
([2]) N. CABIBBO and R. GATTO: *Nuovo Cimento*, **13**, 1086 (1959).
([3]) J. IIZUKA: *Prog. Theor. Phys.* (*Kyoto*), **26**, 554 (1962).
([4]) The dimensionless variable $z=s/m_p^2$ (m_p is the proton mass) ranges from 0 to -0.147 or from -0.013 to -0.147 for the K_{e3} and $K_{\mu 3}$ decays, respectively.

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Consider the process of kaon production by high-energy neutrinos or antineutrinos from a nucleon,

(3) $$\nu + \mathcal{N} \to \mathcal{N} + l^- + K ,$$

(4) $$\bar{\nu} + \mathcal{N} \to \mathcal{N} + l^+ + K ,$$

without specifying the charge channel. The reactions may take place in two ways, namely via production by the baryon core or by the meson cloud. If the pole diagrams are assumed to be dominant, the mechanisms shown in Fig. 1, 2 and 3 play the principal rôle. The core diagram of Fig. 1, however, does not exist in the reactions (1) and (2), because of charge conservation at the weak vertex and of strangeness conservation at the strong vertex respectively. The core diagram of Fig. 2 is supposed to be of minor importance because of the appearance of a heavy virtual particle. Besides, the effective coupling of ($\mathcal{N}$YK) is smaller than that of ($\mathcal{N}\mathcal{N}\pi$) in Fig. 3, although the effective couplings of (Y$\mathcal{N}$lν) and (Kπlν) are of the same order. Thus the peripheral diagram of Fig. 3 is most important and is naturally associated with the leptonic decay diagram of the kaon.

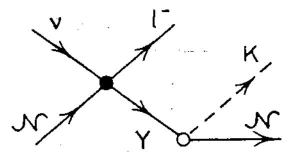

Fig. 1. – The pole diagram for the production in the baryon core (core diagram). The white circle represents the strong interaction and the black the weak interaction.

The reaction of charged kaon production by antineutrinos,

(5) $$\bar{\nu} + p \to p + l^+ + K^- ,$$

(6) $$\bar{\nu} + n \to n + l^+ + K^- ,$$

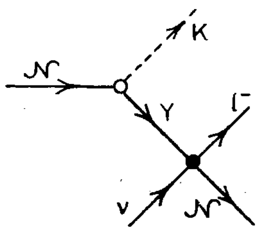

Fig. 2. – The pole diagram for the production in the baryon core (core diagram).

require both diagrams in Fig. 1 and in Fig. 3, but again the former is less important. The peripheral diagram gives the cross-sections for the reaction (5) and (6) identical to those of the reactions (1) and (2) because of the reality properties of the decay form factors, that follow from time-reversal invariance.

Similarly in the K^0 production processes,

$$\nu + n \to p + l^- + K^0 ,$$

$$\bar{\nu} + p \to n + l^+ + K^0 ,$$

the core diagram in Fig. 1 is not allowed by strangeness conservation at the strong vertex, and in the $\overline{K^0}$ production processes,

$$\nu + n \rightarrow p + l^- + \overline{K^0}\,,$$

$$\bar{\nu} + p \rightarrow n + l^+ + \overline{K^0}\,,$$

the core diagram in Fig. 2 is not allowed. However, the peripheral diagrams for these neutral kaon production processes call for the knowledge of the leptonic decay form factors of the neutral kaon,

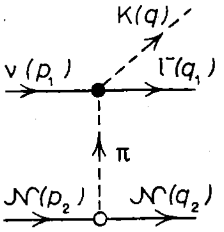

Fig. 3. – The pole diagram for the production in the meson cloud (peripheral diagram). The quantities inside the bracket stand for the four-momentum of the particle.

$$K^0 \rightarrow \pi^- + l^+ + \nu\,,$$

$$K^0 \rightarrow \pi^+ + l^- + \bar{\nu}\,,$$

which are less known than those of the charged kaon.

In this paper we present the cross-section formula of the reaction (1) based on the peripheral mechanism and the expression for the decay rate of K^+_{l3} for arbitrary form factors (Section **2**). By adopting the specific form factors derived from the pole approximation in dispersion theory we give numerical estimates of the cross-section in view of future experiments (Section **3**).

2. – The threshold neutrino energy in the laboratory system of the reaction (1) is 0.79 GeV and 0.62 GeV for the muon and electron mode respectively. The standard manipulation of the peripheral diagram of Fig. 3 [5,6] gives the partial cross-section

$$(7)\qquad \frac{d^3\sigma}{dx\,dy\,dz} = \sigma_0 \frac{1}{(\xi-1)^2}\,\frac{y}{(x+y)(y+\mu)^2}\big[|f(z)|^2\big((x-\lambda)(x+y-z-\lambda)-\varkappa(x+y)\big)+ 2\,\mathrm{Re}\,\big(f(z)g^*(z)\big)\cdot\lambda(x+y-z-\lambda)+|g(z)|^2\cdot\lambda(z+\lambda)\big]\,,$$

where x, y, z, are dimensionless invariant variables

$$x = \frac{-(q_1+q)^2}{m_p^2}\,,\qquad y=\frac{(p_2-q_2)^2}{m_p^2}\,,\qquad z=\frac{(p_1-q_1)^2}{m_p^2}\,,$$

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(5) F. SALZMAN and G. SALZMAN: *Phys. Rev. Lett.*, **5**, 377 (1960); *Phys. Rev.*, **125**, 1703 (1962).

(6) E. FERRARI and F. SELLERI: *Peripheral Model for Inelastic Processes*, CERN Preprint (March 1962). This paper includes a complete bibliography on the subject.

λ, μ, $\varkappa$ are numerical parameters of the masses

$$\lambda = \left(\frac{m_l}{m_p}\right)^2, \qquad \mu = \left(\frac{m_\pi}{m_p}\right)^2, \qquad \varkappa = \left(\frac{m_K}{m_p}\right)^2,$$

and ξ is a dimensionless parameter which specifies the incident energy,

$$\xi = \left(\frac{W}{m_p}\right)^2,$$

W being the total energy in the overall barycentric system related to the incident neutrino energy in the laboratory system E_ν by

$$E_\nu = \frac{W^2 - m_p^2}{2m_p}.$$

The factor σ_0 is defined by

$$\sigma_0 = \frac{1}{32\pi^2} \cdot \frac{g^2}{4\pi} \cdot G^2 m_p^4 \cdot \frac{1}{m_p^2} = 21.2 \cdot 10^{-40}\ \text{cm}^2,$$

where g is the renormalized neutral pion-proton coupling constant ($g^2/4\pi \sim 15$) and G is the Fermi coupling constant ($G^2 m_p^4 \sim 1.0 \cdot 10^{-10}$). The functions f and g are dimensionless form factors that appear in the matrix elements of the kaon-pion current

$$\langle \pi^0 | j_\mu | K^+ \rangle = \frac{1}{(2\pi)^3} \frac{1}{\sqrt{2\omega_K}} \frac{1}{\sqrt{2\omega_\pi}} \left[\frac{f}{2}(p_K + p_\pi)_\mu + \left(\frac{f}{2} - g\right)(p_K - p_\pi)_\mu\right], \tag{8}$$

where p and ω stand for the four-momentum and energy of the indicated meson, and are functions of $s = (p_K - p_\pi)^2$. In the peripheral diagram they are functions of z by the momentum conservation at the weak vertex,

$$\frac{(p_K - p_\pi)^2}{m_p^2} = \frac{(p_1 - q_1)^2}{m_p^2} = z.$$

Specializing eq. (7) for the electron mode we put $\lambda = 0$ and obtain

$$\frac{d^3\sigma}{dx\,dy\,dz} = \sigma_0 \frac{1}{(\xi - 1)^2} \cdot \frac{y[(x - \varkappa)(x + y) - xz]}{(x + y)(y + \mu)^2} |f(z)|^2, \tag{9}$$

which enables us to measure the form factor $f(z)$ directly as a function of z from the angular distribution of the electron under the condition of fixed x and y, if the experimental data are copious enough to allow such a measurement.

The other partial cross-sections $d^2\sigma/dx\,dy$, $d\sigma/dx$ and the total cross-section $\sigma(\xi)$ are obtained by integrating eq. (7) with respect to z, y, x in succession between the limits

$$(10)\qquad z_{\substack{\max\\ \min}}(x, y) = -\lambda + \frac{(x+y)(x-\varkappa+\lambda)}{2x} \pm \frac{x+y}{2x}\cdot\sqrt{x^2-2x(\varkappa+\lambda)+(\varkappa-\lambda)^2}\,,$$

$$(11)\qquad y_{\substack{\max\\ \min}}(x) = \frac{1}{2\xi}\left[(\xi-1)^2-(\xi+1)x \pm (\xi-1)\sqrt{x^2-2x(\xi+1)+(\xi-1)^2}\right],$$

$$(12)\qquad x_{\max}(\xi) = (\sqrt{\xi}-1)^2\,, \qquad x_{\min} = (\sqrt{\varkappa}+\sqrt{\lambda})^2\,.$$

With the form factors defined in eq. (8) the K^+_{l3} decay rate becomes

$$(13)\qquad w(K_{l3}) = w_0\int\limits_{\beta}^{(1-\sqrt{\alpha})^2}\sqrt{\zeta^2-2\zeta(1+\alpha)+(1-\alpha)^2}\left(1-\frac{\beta}{\zeta}\right)^2\cdot$$
$$\cdot\left[|f(\zeta)|^2\left\{\frac{1}{6}\left(\zeta^2-2\zeta(1+\alpha)+(1-\alpha)^2\right)+\beta\right\}\left(1-\frac{2\beta}{\zeta}\right)-\right.$$
$$\left.-\operatorname{Re}\left(f(\zeta)g^*(\zeta)\right)\beta(\zeta+1-\alpha)+|g(\zeta)|^2\cdot\beta\zeta\right]d\zeta\,,$$

where α, β are numerical parameters

$$\alpha = \left(\frac{m_\pi}{m_K}\right)^2, \qquad \beta = \left(\frac{m_l}{m_K}\right)^2,$$

w_0 is given by

$$w_0 = \frac{G^2 m_K^5}{128\pi^3} = 1.48\cdot 10^9\ \text{s}^{-1}\,,$$

and the argument of the form factors is related to the integration variable ζ by

$$\frac{(p_K-p_\pi)^2}{m_K^2} = -\zeta\,.$$

3. – In general the kaon-pion current consists of two currents with definite isospin character $I=\frac{3}{2}$ and $I=\frac{1}{2}$. For each component current a set of form factors f and g is defined by eq. (8), and they are assumed to satisfy certain dispersion relations. If all the intermediate states but the one kaon plus one pion state are neglected, the dispersion relations permit an approximate solution of the form factors in terms of the s- and p-wave phase shifts of the kaon-pion scattering in that particular isospin channel [1-3]. The form factors take a particularly simple form if the kaon plus pion state is dominated by a resonance (pole approximation). Such a resonance state may be associated with the experimentally observed K* particle at 885 MeV [7]. The K*-particle has $I=\frac{1}{2}$, but its intrinsic angular momentum J is not yet identified; actually two apparently contradicting experiments suggest $J=0$ [8] or $J=1$ [7,9,10].

For the sake of definiteness let us assume the $|\Delta I|=\frac{1}{2}$ selection rule, thus confining the kaon-pion current in $I=\frac{1}{2}$ component only, and the K* resonance either in $J=0$ or 1 state. The form factors then become [3,11]:

i) $J=0$ (s-wave resonance) [12]

$$f(z)=C_0\,,\qquad g(z)=\frac{C_0}{2}\left(1-\frac{\varkappa-\mu}{z+\varrho}\right), \tag{14}$$

ii) $J=1$ (p-wave resonance) [13]

$$f(z)=C_1\frac{\varrho}{z+\varrho}\,,\qquad g(z)=\frac{C_1}{2}\frac{\varrho+\varkappa-\mu}{z+\varrho}\,, \tag{15}$$

where

$$\varrho=\left(\frac{m_{K^*}}{m_p}\right)^2,$$

and C_0 and C_1 are constants which may be determined by the experimental decay rate.

Equation (13) is analytically integrable for the electron mode with the form factors (14) and (15). The decay rate reads

$$w(K_{e3})=w_0C_0^2\cdot 2.38\cdot 10^{-2} \tag{16}$$

$$w(K_{e3})=w_0C_1^2\cdot 2.65\cdot 10^{-2} \tag{17}$$

for the s- and p-wave resonance, respectively. Matching these expressions to

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[7] M. ALSTON, L. W. ALVAREZ, P. EBERHARD, M. L. GOOD, W. GRAZIANO, H. K. TICHO and S. G. WOJCICKI: *Phys. Rev. Lett.*, **6**, 300 (1961).

[8] G. ALEXANDER, G. R. KALBFLEISCH, D. H. MILLER and G. A. SMITH: *Phys. Rev. Lett.*, **8**, 447 (1962).

[9] H. W. CHAN: *Phys. Rev. Lett.*, **6**, 383 (1961).

[10] M. A. BAQI BÉG and P. C. DE CELLES: *Phys. Rev. Lett.*, **6**, 145 (1961) with an erratum *Phys. Rev. Lett.*, **6**, 428 (1961).

[11] J. L. ACIOLI and S. W. MAC DOWELL: *Notas de Fisica* (*Rio de Janeiro*), **8**, no. 9.

[12] J. BERNSTEIN and S. WEINBERG: *Phys. Rev. Lett.*, **5**, 481 (1960).

[13] B. D'ESPAGNAT, J. M. JAUCH and Y. YAMAGUCHI: *Strange Particle Physics*, CERN Lecture Notes (November 1959), p. 325.

the experimental K_{e3}-decay rate [14]

$$w(K_{e3})_{exp} = 4.09\cdot 10^{6}\ s^{-1}$$

we find

$$C_0^2 = 0.117\ , \qquad C_1^2 = 0.105\ .$$

Numerical integration of eq. (13) with the form factors (14) and (15) yields the $K_{\mu 3}$-decay rate

$$w(K_{\mu 3}) = w_0 C_0^2\cdot 1.66\cdot 10^{-2} \tag{18}$$

$$w(K_{\mu 3}) = w_0 C_1^2\cdot 1.74\cdot 10^{-2} \tag{19}$$

for the s- and p-wave resonance respectively, and the comparison with the experimental $K_{\mu 3}$ decay rate [14]

$$w(K_{\mu 3})_{exp} = 3.92\cdot 10^{6}\ s^{-1}$$

gives

$$C_0^2 = 0.160\ , \qquad C_1^2 = 0.153\ .$$

As an average the following estimate is adopted for C_0 and C_1:

$$C_0^2 = 0.14 \qquad \text{or} \qquad C_0 = 0.37\ , \tag{20}$$

$$C_1^2 = 0.13 \qquad \text{or} \qquad C_1 = 0.36\ . \tag{21}$$

Numerical integration of eq. (7) with eqs. (10), (11), (12), (14), (15), (20), (21) yields the total cross-section $\sigma(\xi)$ summarized in Table I and II. For the purpose of comparison the total cross-sections of the « elastic » process

$$\nu + n \to p + e^-$$

and the pion production process

$$\nu + p \to p + e^- + \pi^+$$

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[14] We used the life-time of the kaon $1.22\cdot 10^{-8}$ s and the K_{e3}, $K_{\mu 3}$ branching ratio $(5.0\pm 0.5)\%$, $(4.8\pm 0.6)\%$ respectively, see B. P. ROE, D. SINCLAIR, J. L. BROWN, D. A. GLASER, J. A. KADYK and G. H. TRILLING: *Phys. Rev. Lett.*, **7**, 346 (1961).

TABLE I. – *Total cross-section of the reaction* (1) *in units of* 10^{-40} cm² *for the s-wave* K* *resonance eq.* (14).

E_ν (GeV)	ξ	$\sigma(\nu+p\to p+\mu^-+K^+)$	$\sigma(\nu+p\to p+e^-+K^+)$
1	3.13	$4.2\cdot10^{-3}$	$9.6\cdot10^{-3}$
2	5.26	0.207	0.292
3	7.40	0.723	0.978
5	11.7	2.46	3.29
10	22.3	9.55	12.7

TABLE II. – *Total cross-section of the reaction* (1) *in units of* 10^{-40} cm² *for the p-wave* K* *resonance eq.* (15).

E_ν (GeV)	$\sigma(\nu+p\to p+\mu^-+K^+)$	$\sigma(\nu+p\to p+e^++K^-)$
1	$3.1\cdot10^{-3}$	$5.4\cdot10^{-3}$
2	0.095	0.103
3	0.241	0.252
5	0.542	0.554
10	1.14	1.15

at the corresponding energies are quoted from the literature [15,16] and compiled in Table III.

TABLE III. – *Total cross-section of the elastic and pion production reactions in unit of* 10^{-40} cm².

E_ν (GeV)	$\sigma(\nu+n\to p+e^-)$	$\sigma(\nu+p\to p+e^-+\pi^+)$
1	88.6	1.07
2	83.6	5.16
3	81.0	10.7
5	78.5	20.8
10	76.7	40

We observe that at moderate incident energies sufficiently above the threshold there is a difference of a factor $2\div10$ in the total cross-section for two choices of the form factors. With the approximation of vanishing lepton mass

(15) Y. YAMAGUCHI: *Prog. Theor. Phys.* (*Kyoto*), **23**, 1117 (1960).
(16) N. CABIBBO and G. DA PRATO: *Nuovo Cimento*, **25**, 611 (1962).

the momentum transfer variable is

$$z = \frac{(p_1 - q_1)^2}{m_p^2} = \frac{2E_\nu E_1}{m_p^2}(1 - \cos\theta)\,,$$

where E_1 and θ are the energy and production angle of the lepton in the laboratory system respectively. In high-energy region the incident momentum will be carried away mainly by leptons in the forward direction, therefore as a kinematical upper bound z ranges from 0 to z_{max},

$$z_{max} \sim \left(\frac{E_\nu}{m_p}\right)^2 \theta^2_{max}\,,$$

and the range of z increases rapidly with increasing incident energy, thus making the behaviour of the form factors more marked. For the p-wave resonance eq. (15), $|f(z)|^2$ and $|g(z)|^2$ fall off to 1/10 of their maximum value at $z = 0$ already at $z \sim 2$. It is clear that the larger cross-section predicted by the s-wave resonance form factors comes from the constancy of the function f, while the p-wave resonance form factors behave essentially as cut-off factors.

From eqs. (16), (17), (18), (19) the branching ratio of the muon to electron decay mode reads

$$\frac{w(K_{\mu 3})}{w(K_{e3})} = 0.70 \quad \text{and} \quad 0.66\,,$$

for the s- and p-wave resonance respectively [11]. These values are hardly in good agreement with the experimental finding of 1.0 ± 0.2, but this experimental branching ratio is not yet finally established [17].

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[17] J. D. JACKSON: preprint, University of Illinois (1962).

RIASSUNTO

Si calcola, nell'approssimazione periferica, la sezione d'urto per la seguente reazione di produzione di mesoni K da neutrini di alta energia: $\nu + p \to p + l^- + K^+$. Valori numerici per la sezione d'urto sono ottenuti con l'introduzione di specifici fattori di forma, derivati con metodi dispersivi. Poichè i risultati ottenuti dipendono dal modello dinamico adottato, essi possono servire come criterio per la validità del modello stesso.